EQUIPMENT CRITICALITY INDEX: A DEA-BASED MULTI-CRITERIA APPROACH TO SETTING INVESTMENT PRIORITIES FOR INDUSTRIAL PRODUCTION PROCESSES

Área do trabalho: MC – Multicritério

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Abstract

Models that have been classically developed for decision-making processes related to renewing industrial equipment are aimed at the adherence to long-term replacement goals. Many authors have indeed proposed equipment renewing based on replacement and maintenance costs. One of the first approaches, much in use at present, consists of replacing a piece of equipment with a new one after its useful economic life, that is when the long-term total maintenance cost rate (including the replacement cost) is minimum. The aim of this paper is to propose and demonstrate the use of an index for ranking investments in production equipment in manufacturing companies. The construction of the index takes into account multiple criteria and relies on a data envelopment analytical model. Environmental impacts and quality of production, among other criteria, are modeled through the index. In order to illustrate the use of the index an application to a major Cuban heat interchange battery company is included in the paper.

Keywords: Multi-Criteria Decision Analysis – Data Envelopment Analysis - Efficiency of manufacturing equipment

1. INTRODUCTION

In today’s world, characterized by an increasing level of competition, companies need to modernize their equipment in order to achieve continuous improvements in the quality of their products, as well as a systematic reduction in operation costs. Every year, decisions need to be made by manufacturing enterprises when the capital budget is set, with the shortage of financial resources being one of the main aspects that need to be taken into account. The models that have been classically developed related to this kind of decision are aimed at the adherence to long-term replacement goals. On this subject many authors propose equipment renewing based on replacement and maintenance costs. One of the first approaches, much in use at present, consists of
replacing a piece of equipment with a new one after its useful economic life, that is when the long-term total maintenance cost rate (including the replacement cost) is minimum\(^1,2\).

According to Beichelt,\(^3\) the main disadvantage of this policy is that it fails to take into account the individual deviations of the maintenance cost rate of an individual system with regard to the average development cost. To overcome this obstacle, Beichelt proposes a replacement policy as soon as the total maintenance cost per unit of time reaches or exceeds a given level, which represents a cost by unit of optimal limit time. This case includes the analysis of random variables, giving some examples for different probability distributions\(^4\).

On the other hand, Canada et al\(^5\) suggest replacement in terms of Equivalent Uniform Annual Cost. Prawda\(^6\), in turn, proposes the use of dynamic programming models and Markov processes based on the utilities, operation and replacement costs and equipment stop costs.

The previous models generally have a single criteria character. They are used to determine how much longer equipment should be used before being replaced. This presupposes making decisions for the future based on estimates of study variables, which implies frequent revisions of the estimated data. Consequently, there could be a variation in the decisions already made.

This leads to the need to find short-term decisions and techniques that permit the planning of the capital budget from one year to another. These techniques will be based on the use of more representative and embracing variables on the state and performance of the equipment.

2. PROPOSED INDEX

The proposed index is calculated by an additive function and allows a ranking to be established from the calculations of a set of partial, specific indicators on the performance and state of the equipment. This index is based on the fact that the determination of investment in the equipment requires a multi-criteria approach. The possible interval of the index, presented in [1], ranges from 0 to 1.

The additive function is a weighted, linear combination of factors that are considered important in equipment renewing. These aspects include for every equipment quality of production, equipment efficiency, environmental impact, appropriateness in the industrial process, and technological obsolescence. Such an equation reads as follows:

\[
PIE_i = (1 - Q_i) * (f_{Q_i}) + (1 - e_i) * (f_{e_i}) + RIEI_i * (f_{RIEI_i}) + (1 - PI_i) * (f_{PI_i}) + LTO_i * (f_{LTO_i})
\]

where:

- \(PIE_i\): Priority Index of Equipment i
- \(Q_i\): quality of production indicator of equipment i
- \(e_i\): mean comparative efficiency of equipment i
- \(RIEI_i\): relative indicator of environmental impact of equipment i
- \(PI_i\): pertinence indicator of equipment i
- \(LTO_i\): level of technological obsolescence of equipment i
- \(f_{Q_i}\); \(f_{e_i}\); \(f_{RIEI_i}\); \(f_{PI_i}\); \(f_{LTO_i}\): relative weight of each factor

The ranking establishment requires, first of all, an individual analysis of the indicators, taking into account that if one piece of equipment has one or more indicators with inadmissible values, this equipment will have maximum priority. With this established, when there is more than one piece of equipment in this situation then the smallest value obtained by multiplying the indicator by its relative weight will have priority. The rest of the equipment will be ordered from larger (critical equipment) to the smaller PIE values.
3. DETERMINATION OF THE RELATIVE WEIGHT OF EACH FACTOR

To establish the relative weight of each factor, the matched comparison among indicators outlined by Saaty in his Analytic Hierarchy Process (AHP) method is proposed.

Experts are asked to debate among themselves and to answer the following question: With regard to the establishment of a hierarchy of investment in the equipment, how much more important is attribute X in relation to attribute Y? The numbers assigned to the degrees of importance would then follow Saaty’s fundamental scale.

Even numbers are used to represent intermediate values of preferences. For the inverse comparisons (i.e. Y to X), the reciprocal of the numbers in order of importance are used (p_{ab} = 1/p_{ba}) . This information together with the calculation of the priority vector is registered as in Table I. For the calculation of the vector, first of all, data are standardized by dividing each element by the sum of its respective column (s_{CA}, for example, is the sum of data of the correspondent column to the quality indicator). Afterwards a penultimate column is obtained through the sum of standardized rows (s_{FA}, for example, is the sum of the standardized data of the row correspondent to the quality indicator). Finally, the average of each element is determined (last column), which is the weight of the desired vector.

To calculate the consistency of the experts' judgment, matched matrix comparisons should be multiplied by the vector weights. Each element is taken from the resultant vector of this operation and is divided by its equivalent in the vector weights. The resultant values are averaged to obtain the maximum eigenvalue (\lambda_{\text{max}}).

In the case of this matrix, the consistency rate (CR) is determined as follows:

\[
CR = \frac{(\lambda_{\text{max}} - 5) \times 1.12}{4}
\]

According to Saaty, the judgment is consistent if CR is lesser than 0.10.

4. QUALITY OF PRODUCTION INDICATOR

For the calculation of this indicator, a study should be carried out on the existing technological documentation in order to determine the characteristics of the quality of the product that could be influenced by the equipment. Using this information the quality of production indicator obtained by the equipment is determined as below:

\[
Q_i = \frac{AP_i}{PP_i} \times 100\%
\]

where:

- AP_i: amount of products accepted and produced by equipment i
- PP_i: amount of products produced by equipment i

It is important to point out that the products considered as not acceptable would be those whose defects had been caused by the equipment under analysis. The value of this indicator is found in the interval from 0 to 1.

5. MEAN COMPARATIVE EFFICIENCY

This indicator expresses the mean efficiency of the equipment under study compared to a piece of virtual equipment formed by the inputs and outputs of all the equipment of the area. To calculate this indicator, Data Envelopment Analysis (DEA), an extreme point method, is used in a Cross Evaluation Analysis. This relies on the ability of DEA to be used in order to evaluate the

\footnote{1 All tables can be found at the end of this paper.}
efficiency of a number of producers. What DEA does is indeed to compare each producer with only the “best” producer. The “best” virtual producer is then determined for every real producer. In the situation under analysis a producer is equivalent to a piece of equipment. The first thing to be done is to determine all the inputs (I) and outputs (O) for each piece of equipment. In the first case (inputs), one has to consider such aspects as energy, fuel, salaries, maintenance costs, operation time, etc. The second group evaluates the contribution that the equipment makes to production. It is important to point out that these aspects should be established for the same period of time.

For each piece of equipment a linear programming model like the following will be solved:

Min \( \sum I_i \lambda_k \)

subject to:

\[ I_i \lambda_k \geq \sum_{i=1}^{m} \lambda_k I_{ik} \quad \forall_k \]

\[ O_i \leq \sum_{i=1}^{m} \lambda_i O_i \]

\[ \lambda_i \geq 0 \quad \forall_i \]

\[ E_{ij} \geq 0 \]

where:

- \( I_{ik} \): value of input \( k \) of equipment \( i \)
- \( O_i \): value contributed by equipment \( i \)
- \( \lambda_i \): represents the value contributed to the production by each ion is proposed:

\[ O_i = \frac{1}{2} \sum_{h=1}^{w} V_h \left( \frac{q_{oh}}{q_{oh}} + \frac{t_{ih}}{t_h} \right) \]

where:

- \( V_h \): value of the product \( h \)
- \( q_{oh} \): amount of operations performed by equipment \( i \) to the product \( h \)
- \( q_{oh} \): amount of operations needed to elaborate product \( h \)
- \( t_{ih} \): operation time realized on product \( h \) by equipment \( i \)
- \( t_h \): total processing time of product \( h \)
- \( w \): amount of products \( h \)

Once the efficiencies are calculated, according to the previous model, one determines the weight that each piece of equipment assigns to each input \( (u_{ik}) \) and output \( (v_i) \) with the efficiency previously calculated. In order to do this a linear programming model for each piece of equipment is developed allowing the determination of a vector for each of them.

The following model can be formulated for equipment \( m \):

Min \( \left( V_m \sum_{i=m} O_i - \sum_{k} u_{mk} \sum_{i=m} I_{ik} \right) \)

subject to:

\[ \sum_{k=1}^{z} u_{mk} l_{mk} = 1 \]

\[ V_m O_m - E_{mm} \sum_{k=1}^{z} u_{mk} l_{mk} = 0 \]
With the results of the previous model, the efficiencies of equipment i are calculated using the weight vector of equipment m. For this, the next expression is used:

\[
\sum_{k=1}^{z} U_{mk} l_k \leq 0, \quad \forall i \neq m
\]

\[
U_{mk} \geq 0, \quad V_m \geq 0
\]

where:

\(U_{mk}\) : weight given by equipment m to input k.

This calculation is realized in every two-equipment combination. For easier understanding, results are registered in a cross efficiency matrix shown in Table II. In this matrix, the diagonal is the result of the application of the model with the last row representing the mean comparative efficiency value, which is calculated by the expression:

\[
\bar{e}_m = \frac{\sum_{i=1}^{m} E_{mi}}{m}
\]

The mean comparative efficiency is a value in the interval from 0 to 1, being more favorable as the value draws close to 1 or to 100%. Data included in Table II allows for the computation of the mean comparative efficiency value.

6. RELATIVE INDICATOR OF ENVIRONMENTAL IMPACT

To calculate this indicator, negative equipment outputs are used that could influence the external environment and workers’ health. Some outputs due to their intangibility should be qualitatively estimated by an incidence scale indicated by the experts.

The expressions for this indicator are as follows:

\[
RIEI_i = \frac{IEI_i}{\sum_{i=1}^{m} IEI_i} * 100\%
\]

[7]

\[
IEI_i = \sum_{r=1}^{P} \left[ \frac{OE_{ir}}{AV_r} * IF_r \right]
\]

[8]

where:

\(IEI_i\) : environmental impact indicator of equipment i

\(OE_{ir}\) : environmental output r of equipment i

\(AV_r\) : admissible value for output r

\(IF_r\) : impact factor of output r
The admissible values for each output should be consulted in the norm systems by which they are ruled. The impact factors of each output are then assigned by experts, observing the following:

\[ \sum_{r=1}^{p} IF_r = 1 \]  

The value of this indicator will be found in the interval from 0 to 1, being more favorable when it draws closer to 0.

7. PERTINENCE INDICATOR

This indicator expresses the capacity of the equipment to satisfy the needs of production in the present and immediate future. The proposed expression is:

\[ PI_i = \frac{qta_i}{qte_i} * 100 \% \]  

where:
- \( qta_i \): amount of operations each equipment \( i \) is able to execute.
- \( qte_i \): amount of operations that should be carried out by equipment \( i \)

The value of this indicator will be found in the interval from 0 to 1, being more favorable when it draws closer to 1.

8. LEVEL OF TECHNOLOGICAL OBSOLESCENCE

The level of technological obsolescence compared to the modern technologies available in the market should be given by experts, who will place the equipment on a scale from 0 to 100%, 0 being the most favorable level in this case.

9. CASE STUDY

The modeling procedure described above was applied to a major heat interchange battery company, which is owned by the Cuban Ministry of the Sidero-Mechanical and Electronic Industry.

These batteries are produced by the application of interrelated processes applicable to different products developed in this factory. Seven specialized machines took part in these processes. The priority index of each equipment was calculated.

The determination of each weight factor is shown in Table III. The result of the calculation of the consistency rate was \( CR = 0.074 \), which is less than 0.10. This implies that the matched comparisons are appropriate to determine the weight vector.

The calculation of the Priority Index of Equipment was based on the production data of 933 batteries. In Table IV the inputs and the outputs of each piece of equipment are registered jointly with a brief calculation of the indicator. It is important to point out that in the case of the total production time, the preparation time of the equipment as well as the stop time of the equipment was included.

As Table IV shows, equipment \( X_8 \) was classified with high priority. The mean comparative efficiency and the pertinence index of this equipment played a very important role in this result. For this equipment the causes of the problems were determined according to the calculated indicators. Afterwards, a replacement project was established and successfully carried out.
10. CONCLUSIONS

The main characteristic of the proposed index is its multi-criteria approach, which is in accordance with the objectives of the enterprise. The data needed for the calculations are usually available in manufacturing companies, as well as specialized software that allows for the resolution of the associated linear programming models.

The case study demonstrated the usefulness of the procedure in permitting the better allocation of resources, as well as the detection of problems.

Future research on this study should be aimed at the creation of a software for the frequent calculation of the index.

11. REFERENCES


Table I – Calculation of the weight vector of indicators.

<table>
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<tr>
<th>QI</th>
<th>e</th>
<th>RIEI</th>
<th>PI</th>
<th>LOT</th>
<th>PI LTO</th>
<th>QI e RIEI PI LTO</th>
<th>Σ</th>
<th>Σ / 5</th>
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<td>1 pBA</td>
<td>1 pCA</td>
<td>1 pDA</td>
<td>pEA</td>
<td>pAB/sCA</td>
<td>pAC/sCC</td>
<td>pAD/sCD</td>
</tr>
<tr>
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<td>pBA</td>
<td>pBC</td>
<td>pBD</td>
<td>pBE</td>
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Table II – Cross Efficiency Matrix.
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<th>4</th>
<th>5</th>
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Table III – Calculation of the weight vector of indicators.

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<td>10.2</td>
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<th>PI</th>
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Table IV – Inputs and Outputs of each piece of equipment: Calculation of indicators.

<table>
<thead>
<tr>
<th>Item</th>
<th>Electrical energy (Kw)</th>
<th>Salary cost ($)</th>
<th>Total production time (min)</th>
<th>Maintenance cost ($)</th>
<th>$O_i$</th>
<th>$Q_i$</th>
<th>$e_i$</th>
<th>$RIE_i$</th>
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<td>3004.70</td>
<td>0.999</td>
<td>0.628</td>
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<td>$X_2$</td>
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<td>0.273</td>
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